

Detection of Nonlinearly Distorted OFDM Signals via Generalized Approximate Message Passing

Sergey V. Zhidkov, *Member, IEEE*

Abstract—In this paper, we propose a practical receiver for multicarrier signals subjected to a strong memoryless nonlinearity. The receiver design is based on a generalized approximate message passing (GAMP) framework, and this allows real-time algorithm implementation in software or hardware with moderate complexity. We demonstrate that the proposed receiver can provide more than a 2dB gain compared with an ideal uncoded linear OFDM transmission at a BER range $10^{-4} \div 10^{-6}$ in the AWGN channel, when the OFDM signal is subjected to clipping nonlinearity and the crest-factor of the clipped waveform is only 1.9dB. Simulation results also demonstrate that the proposed receiver provides significant performance gain in frequency-selective multipath channels.

Index Terms—OFDM, nonlinear distortion, iterative decoding, message passing, belief propagation

I. INTRODUCTION

MULTICARRIER modulation in the form of orthogonal frequency division multiplexing (OFDM) or discrete multi-tone (DMT) has recently emerged as a preferred candidate for a wide variety of wired and wireless communication systems. It has been used in wireless local area networks [1], in digital subscriber line services [2], in optical communications [3] and is a major contender for fifth generation mobile networks [4].

One of the major disadvantages of OFDM is the high crest-factor of the time-domain OFDM waveform. To avoid in-band and out-of-band distortions, the OFDM signal requires power amplifiers with large back-offs, which results in reduced power efficiency. Several approaches are used to deal with the crest-factor problem in OFDM systems. The simplest method to reduce the crest-factor of an OFDM signal is to apply memoryless nonlinearity, such as clipping, to the OFDM waveform before amplification in a transmitter. This approach allows significant crest-factor reduction, but introduces unwanted in-band distortion components. Conventional receivers treat the distortion component as a noise and thus they suffer from bit error rate (BER) degradation [5]. Decision-feedback nonlinear distortion compensation schemes [6, 7] can partially compensate for BER degradation; however, due to the error propagation effect, they are only suitable for weak nonlinear distortions.

In [8], the authors theoretically demonstrated that the BER performance of multicarrier transmission subjected to strong memoryless nonlinearity can outperform linear transmission provided that an optimal maximum-likelihood (ML) receiver is used. Unfortunately, an ML receiver for nonlinearly distorted

multicarrier signals with a realistic number of subcarriers has enormous complexity. In [9], several sub-optimal "bit-flipping" receivers that can approach ML-performance were proposed. However, the complexity of such receivers grows exponentially with the number of subcarriers and modulation order, and as a result this approach quickly becomes infeasible even for computer simulations.

In this paper, we propose a practical receiver for multicarrier signals subjected to a strong memoryless nonlinearity. The receiver design is based on a generalized approximate message passing (GAMP) framework [10], and it allows real-time algorithm implementation in software or hardware with moderate complexity. We demonstrate that the proposed receiver can provide more than a 2dB gain compared with an ideal uncoded linear OFDM transmission at BERs $10^{-4} \div 10^{-6}$ in the additive white Gaussian noise (AWGN) channel, when the OFDM signal is subjected to clipping nonlinearity and the crest-factor of the clipped OFDM waveform is only 1.9dB.

II. SYSTEM MODEL

Let us consider the system model depicted in Figure 1. In the transmitter, $N/2 - 1$ complex baseband M -QAM modulation symbols are transformed into the time domain via the real inverse discrete Fourier transform (IDFT) operation and then the cyclic prefix (CP) is added to each N -point signal block. Finally, the signal is passed through a memoryless nonlinearity block. The signal at the receiver input can be expressed as:

$$y_n = \sum_{l=0}^L f(z_{n-l}) h_l + w_n, \quad n = 0, \dots, N-1 \quad (1)$$

where $f(z)$ is a memoryless nonlinear function, $\{h_l\}$ is the channel impulse response with length L , w_n is the AWGN term with zero mean and variance σ_w^2 , and

$$z_n = \Re \left\{ \sqrt{\frac{2}{N}} \sum_{k=1}^{N/2-1} \xi_k \exp \left(j \frac{2\pi n k}{N} \right) \right\}, \quad n = 0, \dots, N-1 \quad (2)$$

where ξ_n , $n = 1, 2, \dots, N/2 - 1$ are the complex M -QAM baseband symbols ($\xi_0 = \xi_{N/2} = 0$ to avoid DC-bias). Note that due to CP insertion $z_{-k} = z_{N-k}$, if $k < L$.

In the sequel, we consider the following clipping nonlinearity model:

$$f(z_n) = \begin{cases} T, & \text{if } z_n > T \\ z_n, & \text{if } -T \leq z_n \leq T \\ -T, & \text{if } z_n < -T \end{cases} \quad (3)$$

where T is a threshold value. Note that nonlinearity is applied to a Nyquist-sampled signal $\{z_n\}$; therefore, all distortion components fall in-band and no out-of band emission is introduced.

In this paper, we consider the real-valued OFDM signal, sometimes called a DMT signal, and Cartesian-type clipping nonlinearity. Nonetheless, the results and main conclusions presented in this paper are also applicable to complex OFDM signals and other types of nonlinearity with the saturation effect.

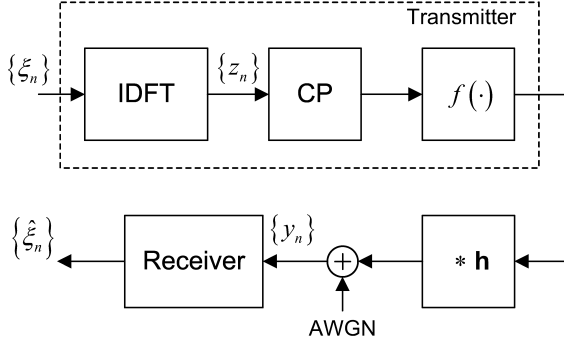


Fig. 1. System model

Assumptions: We assume that (a) the receiver knows the type of nonlinear transfer function $f(z)$, (b) the receiver can perfectly estimate the channel response $\{h_k\}$, and (c) the length of channel impulse response is shorter than the cyclic prefix duration ($L < N_{cp}$).

III. RECEIVER ALGORITHM

A. Case I: AWGN channel

In the AWGN channel ($h_0 = 1$, $h_l = 0$ for $l \neq 0$), the model of the received signal can be simplified to

$$y_n = f(z_n) + w_n, \quad n = 0, 1, \dots, N-1 \quad (4)$$

We can also express $\{z_n\}$ as $\mathbf{z} = \mathbf{F}\mathbf{x}$, where \mathbf{F} is a real $N \times N$ IDFT (unitary) matrix with elements:

$$F_{n,k} = \begin{cases} \sqrt{\frac{2}{N}} \cos\left(\frac{2\pi nk}{N}\right), & 0 \leq k < N/2 \\ -\sqrt{\frac{2}{N}} \sin\left(\frac{2\pi nk}{N}\right), & N/2 \leq k < N \end{cases} \quad (5)$$

and

$$x_n = \begin{cases} \Re(\xi_n), & 0 \leq n \leq N/2 \\ \Im(\xi_{n-N/2}), & N/2 < n < N \end{cases} \quad (6)$$

Model (4) is equivalent to a general problem formulation for the GAMP algorithm [10], which belongs to a class of Gaussian approximations of loopy belief propagation (BP) for dense graphs. In our receiver implementation, we use the sum-product variant of the GAMP algorithm that approximates the minimum mean-squared error (MMSE) estimates of \mathbf{x} and \mathbf{z} . The GAMP algorithm adapted to our problem is summarized

below. The algorithm generates a sequence of estimates $\hat{\mathbf{x}}(t)$, $\hat{\mathbf{z}}(t)$, for $t = 1, 2, \dots$ through the following recursions:

Step 1) Initialization:

$$t = 1, \hat{\mathbf{x}}(1) = \mathbf{0}, \mu^x(1) = \mathbf{1}, \hat{\mathbf{s}}(0) = \mathbf{0}$$

Step 2) Estimation of output nodes:

$$\mu_n^p(t) = \sum_{k=0}^{N-1} |F_{n,k}|^2 \mu_k^x(t), \quad \forall n \quad (7)$$

$$\hat{p}_n(t) = \sum_{k=0}^{N-1} F_{n,k} \hat{x}_k(t) - \mu_n^p(t) \hat{s}_n(t-1), \quad \forall n \quad (8)$$

$$\hat{z}_n(t) = \frac{1}{C} \int_{-\infty}^{\infty} z e^{-\frac{(y_n - f(z))^2}{2\sigma_w^2} - \frac{(\hat{p}_n(t) - z)^2}{2\mu_n^p(t)}} dz, \quad \forall n \quad (9)$$

$$\mu_n^z(t) = \frac{1}{C} \int_{-\infty}^{\infty} z^2 e^{-\frac{(y_n - f(z))^2}{2\sigma_w^2} - \frac{(\hat{p}_n(t) - z)^2}{2\mu_n^p(t)}} dz - (\hat{z}_n(t))^2, \quad \forall n \quad (10)$$

where

$$C = \int_{-\infty}^{\infty} e^{-\frac{(y_n - f(z))^2}{2\sigma_w^2} - \frac{(\hat{p}_n(t) - z)^2}{2\mu_n^p(t)}} dz, \quad \forall n \quad (11)$$

$$\hat{s}_n(t) = (\hat{z}_n(t) - \hat{p}_n(t)) / \mu_n^p(t), \quad \forall n \quad (12)$$

$$\mu_n^s(t) = \frac{1}{\mu_n^p(t)} \left(1 - \frac{\mu_n^z(t)}{\mu_n^p(t)} \right), \quad \forall n \quad (13)$$

Step 3) Estimation of input nodes:

$$\mu_k^r(t) = \left(\sum_{n=0}^{N-1} |F_{n,k}|^2 \mu_n^s(t) \right)^{-1}, \quad \forall k \quad (14)$$

$$\hat{r}_k(t) = \hat{x}_k(t) + \mu_k^r(t) \sum_{n=0}^{N-1} F_{n,k} \hat{s}_n(t), \quad \forall k \quad (15)$$

$$\hat{x}_k(t+1) = \sum_{m=1}^{\sqrt{M}} d_m P_{m,k}, \quad \forall k \quad (16)$$

$$\mu_k^x(t+1) = \sum_{m=1}^{\sqrt{M}} (d_m - \hat{x}_k(t+1))^2 P_{m,k}, \quad \forall k \quad (17)$$

where

$$P_{m,k} = \frac{e^{-\frac{(d_m - \hat{r}_k(t))^2}{2\mu_k^r(t)}}}{\sum_{l=1}^{\sqrt{M}} e^{-\frac{(d_l - \hat{r}_k(t))^2}{2\mu_k^r(t)}}}, \quad (18)$$

M is the number of points in the signal constellation, and $\{d_m\}$ is the vector of constellation points per real or imaginary component, e.g. for 4-QAM modulation $\{d_m\} = [-1 \ 1]$ and for 16-QAM $\{d_m\} = [-3/\sqrt{5} \ -1/\sqrt{5} \ 1/\sqrt{5} \ 3/\sqrt{5}]$.

The steps (7)-(17) are repeated with $t = t+1$ until t_{max} iterations have been performed. At each iteration, we calculate the Euclidean distance between the received vector $\{y_k\}$, and reconstructed time-domain waveform $f\left(\sum_{k=0}^{N-1} F_{n,k}\hat{x}_k(t)\right)$, and the final decision is based on vector $\{\hat{x}_k(t)\}$ that corresponds to the minimum Euclidean distance $E(t)$:

$$E(t) = \sum_{n=0}^{N-1} \left(y_n - f\left(\sum_{k=0}^{N-1} F_{n,k}\hat{x}_k(t)\right) \right)^2 \quad (19)$$

It should be noted that the integrals in (9), (10) and (11) can be expressed in closed-form using the tabulated Gauss error functions. However, due to the complexity of closed-form expressions, we omit them here for clarity. In practice, (9)-(11) can be approximated using several simple look-up tables.

B. Case II: Multipath channel

With the addition of the multipath channel, the system model (1) deviates from a classical GAMP estimation problem. There is an additional "channel" sub-graph connecting nonlinear nodes $\{f(z_n)\}$ and observable nodes $\{y_n\}$. An optimal solution would be to incorporate the channel sub-graph into a message passing receiver structure. However, in this paper, we resort to a sub-optimal, yet computationally simpler approach. Namely, we introduce a channel pre-processing step:

$$\mathbf{y}' = IDFT\left(\mathbf{Y} \circ \mathbf{H}^{\circ(-1)}\right) \quad (20)$$

where \mathbf{Y} is the DFT of \mathbf{y} , \mathbf{H} is the DFT of the channel impulse response \mathbf{h} , \circ denotes Hadamard product, and $\circ(-1)$ denotes Hadamard (element-wise) inversion. After such a pre-processing step, we can apply GAMP algorithm (7)-(17) to the signal $\{y'_n\}$, which now can be expressed as

$$y'_n = f(z_n) + w'_n, \quad n = 0, 1, \dots, N-1 \quad (21)$$

where $\{w'_n\}$ is the Gaussian distributed, but non-white noise sequence. Note that we rely here on our assumption that the channel impulse response length is shorter than the CP duration, which allows us to model the linear convolution of $\{f(z_n)\}$ with the channel impulse response $\{h_k\}$ as a cyclic convolution. One can easily observe that (20) is essentially a well-known zero-forcing linear equalization operation. This approach is clearly sub-optimal since the GAMP receiver operating on $\{y'_n\}$ ignores the correlation of noise components in $\{w'_n\}$. Nonetheless, it permits low-complexity receiver implementation and demonstrates very good performance in typical multipath scenarios, as will be illustrated in the next section.

C. Receiver complexity

One of the advantages of the GAMP-based receiver is its moderate computational complexity. In fact, the complexity of algorithm (7)-(17) is dominated by the matrix transforms in (8) and (15), which can be computed efficiently via fast Fourier transform (FFT). Thus, the complexity of the proposed

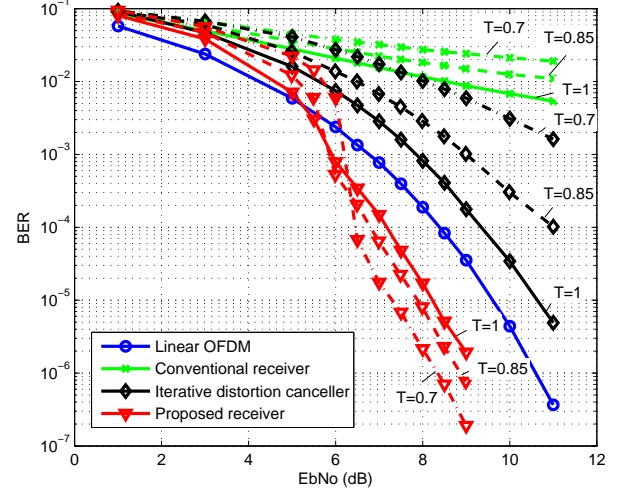


Fig. 2. Performance of the proposed receiver in AWGN channel (4-QAM, $N=4096$)

receiver is of order $O(4tN \log_2(N))$, where t is the number of iterations. This compares favorably with the complexity of maximum-likelihood receiver and sub-optimal receivers in [9]. Overall, the receiver complexity per iteration is comparable with that of the iterative distortion cancellers [6, 7].

IV. SIMULATION RESULTS AND DISCUSSION

The performance of the proposed decoding algorithm has been studied by means of Monte-Carlo simulation. We simulated an uncoded OFDM system with 2048 complex subcarriers ($N=4096$) and 4-QAM and 16-QAM modulation with Gray mapping. The variance of $\{z_k\}$ was normalized to 1. For comparison, we also include here the results for a conventional OFDM receiver and an iterative distortion canceller that attempts to estimate and compensate for the uncorrelated nonlinear distortion term (so called, Busgang noise) using a decision-directed approach [6, 7]. In all our simulations, the maximum number of iterations (t_{max}) was set to 30. However, it should be noted that when the BER is less than 10^{-4} the average number of iterations required for convergence is typically less than 5. The results for the AWGN channel are illustrated in Figures 2 and 3.

As can be seen from the results presented in Figure 3, the clipped 4-QAM OFDM signal without any forward error-correction coding provides 2 ÷ 2.3 dB gain compared with the *ideal* uncoded linear transmission at $BER=10^{-4} \div 10^{-6}$ if the clipping threshold is set to $T = 0.7$ (this value of T corresponds to a clipped waveform crest-factor around 1.9 dB). The performance gain can be as high as 2.5 dB at $BER=10^{-6}$ when $T = 0.65$. At lower clipping ratios ($T \leq 0.6$), the GAMP-based receiver does not converge at practically interesting values of E_b/N_0 . At higher values of the clipping threshold ($T = 0.8 \div 1$) the performance improvement at low BERs is less dramatic, but the water-fall region starts at lower values of E_b/N_0 , which results in slightly better performance at BER around 10^{-3} .

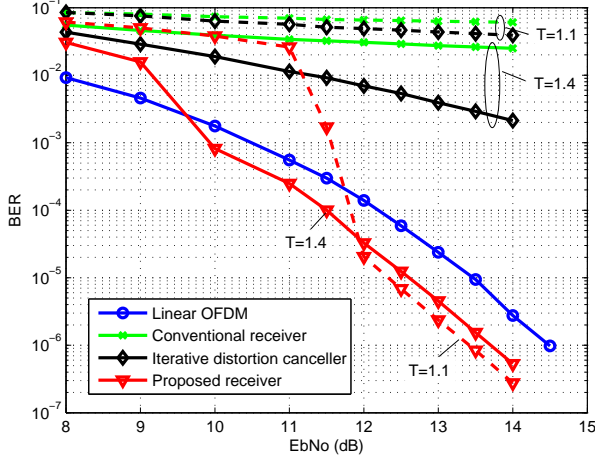


Fig. 3. Performance of the proposed receiver in AWGN channel (16-QAM, $N=4096$)

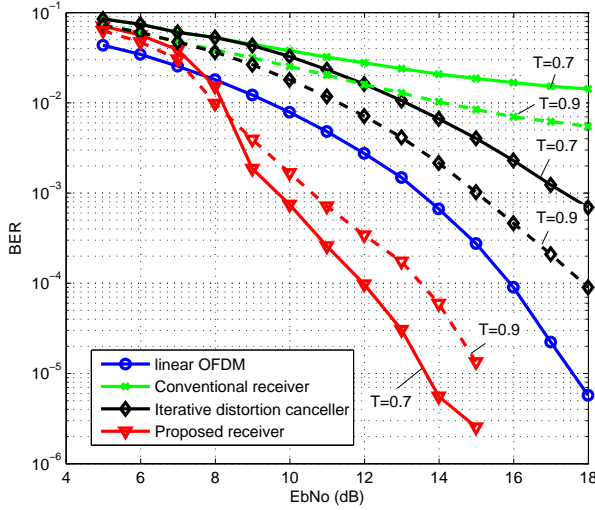


Fig. 4. Performance of the proposed receiver in frequency selective channel (4-QAM, $N=4096$, $T=0.8$)

A less remarkable, but still significant, performance gain (>1 dB at $\text{BER}=10^{-5} \div 10^{-6}$) can also be achieved by a GAMP-based receiver in the case of 16-QAM modulation (see Figure 3). It should be noted, however, that in the 16-QAM case the GAMP-based receiver does not converge at threshold values $T \leq 1$. On the other hand, lower threshold values lead to a better theoretical performance [8], therefore, the convergence of message passing receivers for nonlinearly distorted OFDM signals in the case of very strong nonlinear distortions is still an open issue.

Finally, in Figure 4 we illustrate the performance results for the GAMP-based receiver with a channel pre-processing

step (20) in the frequency-selective multipath channel. The channel profile was generated randomly from distribution $h_k = \mathcal{N}(0, 1) \exp(-0.05k)$, $k = 0, \dots, 63$. The frame length was set to $N = 4096$, the duration of the CP was set to $N_{cp} = 64$. The results show that the proposed receiver can achieve around $3 \div 4$ dB gain in such multipath channels even with sub-optimal channel equalization processing (20).

V. CONCLUSIONS

In this paper, we proposed a practical GAMP-based receiver for multicarrier signals subjected to strong memoryless nonlinearity. We demonstrated that the proposed receiver can provide more than a 2dB gain compared with the *ideal* linear OFDM transmission at a BER range $10^{-4} \div 10^{-6}$ in the AWGN channel. Note that at the same time, the crest-factor of the Nyquist-sampled OFDM waveform is reduced from ~ 12 dB to only 1.9dB as a result of clipping. The simulation results also show that the proposed receiver provides significant performance gain in frequency-selective multipath channels. Due to this fact, we believe that, in some applications, uncoded clipped OFDM systems combined with the proposed receiver structure can compete with coded OFDM systems. The complexity of the proposed receiver is moderate and is suitable for practical implementation in software or hardware. The extension of the proposed receiver structure to coded OFDM systems and the full utilization of the graph structure for severe frequency-selective multipath channels is a possible venue for future research.

REFERENCES

- [1] Local and Metropolitan Area Networks – Specific Requirements – Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications, IEEE Standard 802.11ac, 2013
- [2] Fast access to subscriber terminals (G.fast) – Physical layer specification, Recommendation ITU-T G.9701, 2014
- [3] J. Armstrong, “OFDM for Optical Communications,” J. Lightw. Technol., vol. 27, no. 3, pp. 189-204, Feb. 2009
- [4] J. Andrews, S. Buzzi, W. Choi, S. Hanly, A. Lozano, A. Soong, J. Zhang, “What will 5G be?” IEEE J. Sel. Areas Commun., vol. 32, no. 6, pp. 1065-1082, June 2014
- [5] P. Banelli and S. Cacopardi, “Theoretical analysis and performance of OFDM signals in nonlinear AWGN channels,” IEEE Trans. Commun., vol. 48, no. 3, pp. 430-441, Mar. 2000
- [6] J. Tellado, L. M. C. Hoo and J. M. Cioffi, “Maximum-likelihood detection of nonlinearly distorted multicarrier symbols by iterative decoding,” IEEE Trans. Commun., Vol. 51, No. 2, pp. 218-228, Feb. 2003
- [7] H. Chen and A. M. Haimovich, “Iterative estimation and cancellation of clipping noise for OFDM signals,” IEEE Commun. Lett., vol. 7, no. 7, pp. 305-307, July 2003
- [8] J. Guerreiro, R. Dinis and P. Montezuma, “On the Optimum Multicarrier Performance With Memoryless Nonlinearities,” IEEE Trans. Commun., vol. 63, no. 2, pp. 498-509, Feb. 2015
- [9] J. Guerreiro, R. Dinis and P. Montezuma, “Optimum and Sub-Optimum Receivers for OFDM Signals with Strong Nonlinear Distortion Effects,” IEEE Trans. Commun., vol. 61, no. 9, pp. 3830-3840, Sept. 2013
- [10] S. Rangan, “Generalized approximate message passing for estimation with random linear mixing,” in Proc. IEEE Int. Symp. Information Theory, Saint-Petersburg, Russia, 2011, pp. 2174-2178